# Climate Models and the Irrelevance of Chaos

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# Hawkmoths, Butterflies, and the Epistemology of Climate Models

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# Intro

# Prediction

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- Start with **initial conditions**.
- Apply **dynamical laws**.
- Generate (expected) **outcomes**.

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# Prediction

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- Start with **initial conditions**.
- Apply **dynamical laws**. $\Leftarrow$ minimal uncertainty
- Generate (expected) **outcomes**. $\Leftarrow$ substantial uncertainty

Others exhibit **SDDL**: sensitive dependence on dynamical laws.

# The controversy

The “LSE group” (allegedly):

1. SDDL is **worse** than SDIC.
2. (And thus) uncertainty about dynamical laws is **worse** than uncertainty about initial conditions.

# Enter this talk

Is dynamical uncertainty worse **in principle**? No.

(So long as there are only countably many possibilities.)

Is dynamical uncertainty worse **in practice**? Probably.

# Plan

1. The demon argument.
2. In principle: equally bad.
3. In practice: dynamical is worse.

# The demon argument

# Laplace's demon

Imagine a perfect calculator (Frigg et al. 2014):

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What if we're uncertain about the IC?

$$\textit{pr(IC)} + \textit{true DL} \Rightarrow \textit{pr(outcomes)}$$

**Important**: if $pr(IC)$ is "calibrated" so too is $pr(outcomes)$. (Even if the system exhibits SDIC!)

# Introducing dynamical uncertainty

Dynamical uncertainty causes problems:

$$pr(IC) + best\ guess\ DL \Rightarrow pr(outcomes)$$

If the system exhibits SDDL, doesn't matter how good the "best guess" is, the outcomes can be arbitrarily inaccurate.

# What does this show?

(1) SDDL and SDIC *together* are worse than SDIC alone.

(2) Uncertainty about both laws and initial conditions is worse than uncertainty about just initial conditions.

**But not**:

(3) SDDL alone is worse than SDIC alone.

(4) Uncertainty about laws is worse than uncertainty about initial conditions.

# An in-principle comparison

# Return of the demon argument

Demon 1 has a probability distribution over initial conditions, but knows the dynamics.

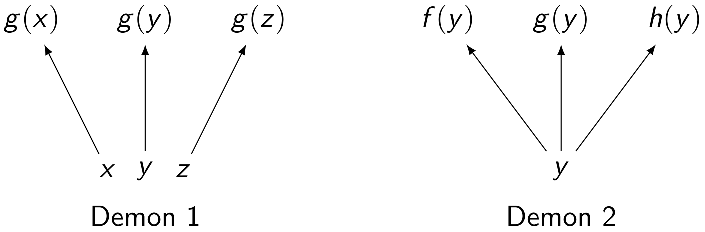
Demon 2 has a probability distribution over the dynamics, but knows the initial conditions.

# All other things being equal

(To make life easy: for each initial condition recognized by demon 1, there is exactly one dynamical arrangement recognized by demon 2.) Then:

- the relevant initial conditions and dynamics lead to equally wrong errors in outcomes
- the two demons assign equal probability to the relevant initial conditions and dynamical arrangements

# Example



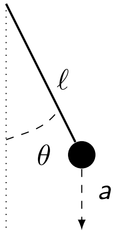
# An in-practice comparison

# Identical strategies

Demon 1: considers every possible set of initial conditions, evolves them according to the true laws, generates outcomes.

Demon 2: begins with the true initial conditions, evolves them according to every possible set of laws, generates outcomes.

# A simple example




- $\ell$ and $a$ are constants.
- $t$ is conventional.
- Possible initial conditions: subset of $\mathbb{R}$ (0 - 360°).
- Possible dynamics: *much* more complicated.

# Possible dynamics

"True" dynamics:

$$\theta_t = \theta_0 \cos\left(\sqrt{\frac{a}{\ell}}t\right)$$

Two possible alternatives:

$$\theta_t = \theta_0 \cos\left(\sqrt{\frac{a}{\ell}}\right)$$

$$\theta_t = \theta_0 - \frac{2}{3}\left(\sqrt{\frac{a}{\ell}}t\right)$$

# Still...

This contrast is almost certainly too stark.

- Things gets more complicated for initial conditions when we don't know which variables matter.
- Our understanding of initial conditions often depends on the our understanding of the dynamics.

# Fin

Thank you!

And if you want to read the original paper, "Climate Models and the Irrelevance of Chaos" (Dethier forthcoming), it's on my website (coreydethier.com).

# Importance for climate models

1. General arguments regarding SDDL and error in climate modeling stand or fall with general arguments regarding SDIC and error (compare Dethier forthcoming).
2. We shouldn't expect strategies that are efficient for addressing SDIC will be efficient at addressing SDDL.
3. In Dethier (forthcoming), I argue that the LSE group's arguments indicate that dynamical uncertainty may be particularly intractable. This reinforces that conclusion.

# Defining SDIC

**SDIC**: $\langle f, X, t \rangle$ exhibits SDIC iff for all $x \in X$ there is some $y$ "arbitrarily close" to $x$ such that

$$d(y_t, x_t) > e^{\lambda t} d(y, x)$$

Compare (Mayo-Wilson 2015).

# A parallel SDDL

First, a definition of distance measure $D$:

$$D(f, g, x, i) =_{def} d(g_i(x), f_i(x))$$

**SDDL**: $\langle F, x, t \rangle$ exhibits SDDL iff for all $f \in F$ there is some $g$ "arbitrarily close" to $f$ such that

$$D(f, g, x, t) > e^{\lambda t} D(f, g, x, i)$$

for some $i < t$.

# The demon

Given a probability distribution $Pr$ over possible values for initial conditions $x$, the demon fixes their probability distribution over possible states at time $t$ so that

$$Pr(x_t = a) = Pr(f_t(x) = a)$$

Notice: the same equation works when the demon's probability distribution ranges over possible functions rather than initial conditions (or, indeed, both).

# All other things equal

Formally: for every $x$ that demon one deems indistinguishable from the true $y$ there is some $f$ that computer two deems indistinguishable from the true $g$ s.t.

$$d(g_t(x), g_t(y)) = d(f_t(y), g_t(y))$$

for the given $t$ and

$$Pr_1(y = x) = Pr_2(g = f)$$

where $Pr_1$ and $Pr_2$ are the probabilities that the two demons assign to the relevant propositions.

# Uncountable cases

Introduce a problem for uncertainty over laws.

Roughly: while there's always a canonical way to measure the relative "sizes" of infinite regions of a state space, the same isn't true for sets of dynamical laws. (Recall the pendulum equations.)

Upshot: sometimes we can't define a probability density function over functions in a non-arbitrary way.

# Uncountable cases, cont'd

That's not to say that we *never* can. Consider:

$$\theta_t = \theta_0 \cos\left(\sqrt{\frac{a}{l}}\right) \tag{1}$$

$$\theta_t = \theta_0 \cos\left(\sqrt{\frac{a}{\ell}}t\right) \tag{2}$$

Generalizing:

$$\theta_t = \theta_0 \cos\left(\sqrt{\frac{a}{\ell}}t^i\right)$$

$$Pr[(1) \preceq X \preceq (2)] = \int_0^1 f_X(i)di$$

- Dethier, Corey (forthcoming). Climate Models and the Irrelevance of Chaos. *Philosophy of Science.*
- Frigg, Roman et al. (2014). Laplace's Demon and the Adventures of His Apprentices. *The Journal of Philosophy* 81.1: 31–59.
- Mayo-Wilson, Conor (2015). Structural Chaos. *Philosophy of Science* 82.5: 1236–47.